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BRIEF NOTE

INITIAL PRESSURE DISTRIBUTION DUE TO JET IMPACT ON A RIGID BODY

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The analysis in this paper shows that, after an impulse due to a two-dimensional jet having velocity U and density ρ hitting a rigid body, the initial pressure distribution over the wall has the constant value ρU^2 relative to the ambient pressure. It also reveals that a discontinuity exists in the pressure at the intersection of the surface of the body and the surface of the jet. These results have been confirmed by a numerical solution based on a boundary element method.

1. DESCRIPTION OF THE PROBLEM

We consider the problem, of a flat-headed jet with velocity U and thickness $2d$ moving towards a rigid wall. When the jet hits the surface of the body, shown in Figure 1, it might seem plausible to assume that the pressure over the wall will vary from the ambient pressure at the intersection point A to a higher value at the centre-point O of the jet. Although it is less convincing, one may also speculate that the pressure across a thin jet near the wall should be equal to the ambient pressure, namely the pressure away from the wall. What is observed in this paper, however, does not entirely follow these hypotheses. The problem presented here is dynamically equivalent to the case where the fluid is at rest and is in contact with the solid surface which starts moving suddenly with velocity U. As argued by Wu (1998), for this kind of problem associated with sudden motion, the initial impact can be divided into two stages: (i) the impulsive stage between $0_- \leq t < 0_+$, and (ii) the post-impulse stage $t = 0_+$. The results within these two stages are quite different, as shown in what follows.

2. THE IMPULSIVE STAGE

As stated in Section 1, the jet impact problem is dynamically equivalent to the problem of a rigid plate moving suddenly against the liquid which was at rest. The viscous effect is not important during the initial stage of the impact, as the spatial gradients are negligible in comparison with the time derivative (Batchelor 1967, p. 471). Thus, the problem can be solved based on the assumptions of potential flow. Due to symmetry, the centreline of the jet can be treated as a rigid surface, as shown in Figure 2. The governing equation and boundary conditions can then be given as

$$\nabla^2 \phi = 0 \tag{1}$$

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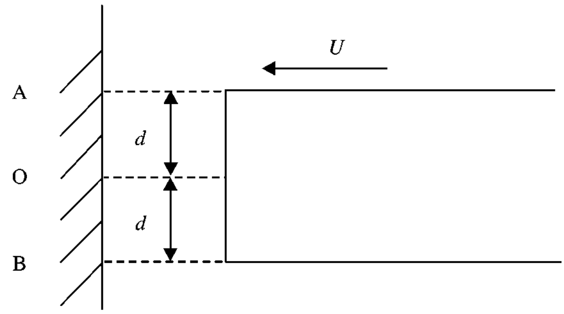

Figure 1. Sketch of the problem.

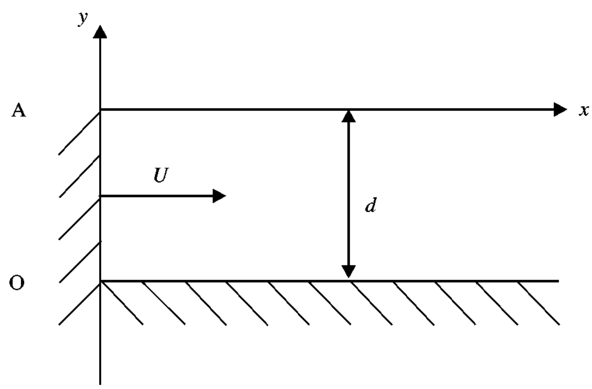

Figure 2. Computational model.

in the fluid domain;

$$\frac{\partial \phi}{\partial x} = U, \quad x = 0, \tag{2}$$

$$\frac{\partial \phi}{\partial y} = 0, \quad y = -d, \tag{3}$$

$$\phi = 0, \quad x \to \infty, \tag{4}$$

$$\phi = 0, \quad y = 0. \tag{5}$$

The last equation can be obtained following the argument in Batchelor (1967, p. 473).

The solution for this problem can be found quite easily, see Peregrine (1972), for example. It can be written as

$$\phi = \frac{2U}{d} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{k_n^2} \mathrm{e}^{-k_n x} \cos k_n (y + d), \tag{6}$$

where $k_n = (n\pi + \pi/2)/d$. The pressure distribution could then be obtained from the Bernoulli equation

$$p = -\rho\phi_t - \tfrac{1}{2}\rho \nabla\phi \cdot \nabla\phi, \tag{7}$$

where ρ is the density of the liquid and the subscript t indicates derivative, and zero ambient pressure has been assumed. Because the time period over which the impact occurs is zero, the pressure will be infinite. But what is of practical interest in this case is the pressure impulse defined as (Batchelor 1967, p. 471)

$$\Pi = \int_{0_-}^{0_+} p \, \mathrm{d}t = -\rho\phi. \tag{8}$$

This equation shows that the variation of the impulse follows that of the potential, which varies from zero at $y = 0$ to a maximum value at $y = -d$ along the wall (see Table 1). The result is therefore consistent with the first hypothesis in the introduction.

3. PRESSURE DISTRIBUTION IMMEDIATELY AFTER THE IMPULSE

Immediately after the impulsive stage, the time derivative is no longer infinite. Both terms on the right-hand side of equation (7) should be included in the calculation of pressure. At $t = 0_+$, the potential itself is the same as that given in equation (6) but ϕ_t becomes unknown. Note that ϕ_t satisfies the Laplace equation. On $y = 0$, the Bernoulli equation gives

$$\phi_t = -\tfrac{1}{2}\nabla\phi \cdot \nabla\phi. \tag{9}$$

On the solid wall, we have (Wu 1998)

$$\frac{\partial \phi_t}{\partial x} = -U\frac{\partial^2 \phi}{\partial x^2}. \tag{10}$$

The other boundary conditions on ϕ_t are the same as those on ϕ. It would then be possible to treat ϕ_t as another potential which could be derived in a manner similar to that used to obtain ϕ. The procedure would be straightforward. But even that is not necessary for this particular configuration. In fact, it can be confirmed that

$$\phi_t = -\tfrac{1}{2}(\phi_y^2 - \phi_x^2) - 2U\phi_x \tag{11}$$

satisfies all the boundary conditions and the Laplace equation, despite the product terms. The Bernoulli equation becomes

$$p = 2\rho U\phi_x - \rho\phi_x^2. \tag{12}$$

Using equation (2), the pressure on the body surface can be found as

$$p = \rho U^2, \tag{13}$$

which is a constant. This is a quite surprising result and it shows that neither of the hypotheses in Section 1 is valid at this stage of the impact. The result also shows that the pressure is discontinuous at the intersection point A; it equals ρU^2 if the point is approached from the solid surface, and equals zero if the point is approached along $y = 0$. The contradiction is clearly due to the incompatibility of the conditions on ϕ in equations (2) and (4). It is well known that this incompatibility leads to an infinitely large vertical velocity at the intersection and in a numerical analysis (e.g. Lin *et al.* 1985) some suitable treatment is needed in order to obtain a realistic solution for the flow near the intersection. The discontinuity in pressure caused by the incompatibility revealed here seems not to have been observed before.

To verify equation (13), the boundary value problems for ϕ and ϕ_t have been solved numerically, using a boundary element method (Wu & Eatock Taylor 1995). The length of

TABLE 1
Potential along the body surface

y/d	ϕ/Ud ($s/d = 0{\cdot}01$)	ϕ/Ud ($s/d = 0{\cdot}02$)	ϕ/Ud ($s/d = 0{\cdot}04$)	Equation (6)
0·00	0·000	0·000	0·000	0·000
− 0·01	− 0·037			− 0·038
− 0·02	− 0·066	− 0·066		− 0·065
− 0·03	− 0·091			− 0·091
− 0·04	− 0·114	− 0·114	− 0·114	− 0·113
− 0·05	− 0·135			− 0·135
− 0·06	− 0·155	− 0·155		− 0·155
− 0·07	− 0·174			− 0·173
− 0·08	− 0·192	− 0·192	− 0·192	− 0·192
− 0·09	− 0·209			− 0·209
− 0·10	− 0·226	− 0·226		− 0·226
− 0·11	− 0·241			− 0·241
− 0·12	− 0·257	− 0·257	− 0·257	− 0·257
− 0·13	− 0·272			− 0·272
− 0·14	− 0·286	− 0·286		− 0·286
− 0·15	− 0·300			− 0·300
− 0·16	− 0·313	− 0·313	− 0·313	− 0·313
− 0·17	− 0·326			− 0·326
− 0·18	− 0·339	− 0·339		− 0·338
− 0·19	− 0·351			− 0·351
− 0·20	− 0·363	− 0·363	− 0·363	− 0·363

the computational domain is taken as $10d$. As the main concern here is not computational efficiency, uniform elements of length s have been used over the boundary. Tables 1 and 2 give the potential and pressure, respectively, along the body surface, when different values of s are used. Results are provided between $y = 0$ to $-0.2d$ to highlight their behaviour in this region of interest. It can be seen that the numerical solutions agree well with the results in equations (6) and (13), apart from the pressure near the intersection. This discrepancy is due to the discontinuity.

Using the numerical method, it is possible to analyse the jet deformation based on the time-marching technique. If the flow field is the main interest in the analysis, the observed discontinuity in the pressure should not cause too much concern, because the pressure is a product obtained after the potential has been found. It has no feedback to the flow if the body is rigid and fixed. In other cases, such as an elastic plate or a nonfixed rigid body, correct prediction of pressure is an essential part of the analysis, as observed by Lu, *et al.* (2000). An error in the pressure will lead to a false response of the body, which will in turn give a false feedback to the flow. It is quite possible that the error will accumulate with time, and the numerical solution will depart from reality, with instabilities possibly occurring. Thus, proper understanding of the pressure behaviour is extremely important in the development of numerical analyses. Table 2 shows that the nondimensionalized pressure is near unity everywhere else, but behaves rather erratically near the intersection. This kind of behaviour would be almost certain to cause concern to the numerical analyst. Equation (13) gives a clear indication that such behaviour is not abnormal. Indeed, Table 2 shows that the region in which the erratic behaviour can be observed becomes smaller as the element size is reduced. It also shows that at a given point below $y = 0$, the result does indeed tend to one as s is decreased. All these observations are consistent with what is found in this paper.

TABLE 2
Pressure along the body surface

y/d	$p/\rho U^2$ ($s/d = 0{\cdot}01$)	$p/\rho U^2$ ($s/d = 0{\cdot}02$)	$p/\rho U^2$ ($s/d = 0{\cdot}04$)
0·00	0·000	0·000	0·000
− 0·01	− 0·257		
− 0·02	0·842	− 0·114	
− 0·03	0·920		
− 0·04	0·954	0·868	0·029
− 0·05	0·970		
− 0·06	0·979	0·933	
− 0·07	0·985		
− 0·08	0·988	0·962	0·895
− 0·09	0·990		
− 0·10	0·992	0·975	
− 0·11	0·993		
− 0·12	0·994	0·983	0·946
− 0·13	0·995		
− 0·14	0·996	0·987	
− 0·15	0·996		
− 0·16	0·996	0·990	0·969
− 0·17	0·997		
− 0·18	0·997	0·992	
− 0·19	0·997		
− 0·20	0·998	0·993	0·980

4. CONCLUDING REMARKS

The main result obtained in this paper is the pressure distribution at $t = 0_+$, immediately after an impulse. The discontinuity in pressure is due to the incompatibility of the conditions for the potential at the intersection. $\phi = 0$ used on $y = 0$ is a result of the product term in the Bernoulli equation being ignored during the impulse, based on the assumption that the spatial gradients are much smaller than the time derivative during the impulse. Equation (6) shows that such an assumption is valid everywhere apart from at the intersection. In fact, at point A, $\varphi_y \to \infty$ and one cannot automatically assume that $\varphi_t \gg \varphi_y^2$. Because these two terms tend to infinity in the temporal and spatial domains, respectively, it is not straightforward to compare their relative magnitudes. It is therefore important to emphasize that the discontinuity noticed in this paper may be a result of the mathematical model. Further investigation is clearly needed, especially some experimental work, similar to that by Chan (1994) and Smith *et al.* (1998).

The significance of what has been discussed here may mainly lie in the implications for numerical simulation. Some treatment for the pressure shown in Table 2 is clearly needed, if one is going to use it to calculate the plate deformation. In this context, the result given in equation (13) would be valuable. However, this paper does not offer any real contribution towards resolving the singularity mathematically, such as by introducing an inner solution. This is, on the other hand, not the motivation of this work.

It is also worth pointing out that, in reality, impact may start from $t = t_b$ and end at $t = t_a$. In his paper, Cooker (1996) offered a model for $t_a - t_b \neq 0$, which included both the spatial and temporal derivatives in the Bernoulli equation when the dynamic condition is imposed on $y = 0$. But it is fair to say that it is still not clear whether his model reflects physical reality.

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